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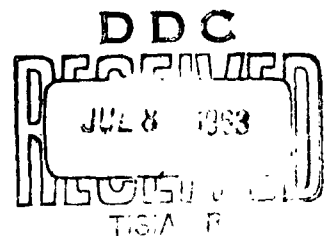
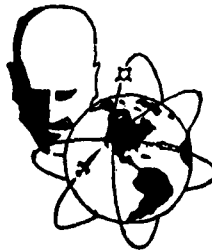
**TAPPED DELAY LINE SYNTHESIS OF LARGE TIME-BANDWIDTH SIGNALS**

**TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-63-180**

June 1963

**R. D. Haggarty**

Prepared for  
**DIRECTORATE OF RADAR AND OPTICS  
ELECTRONIC SYSTEMS DIVISION  
AIR FORCE SYSTEMS COMMAND  
UNITED STATES AIR FORCE  
L. G. Hanscom Field, Bedford, Massachusetts**



Prepared by  
**THE MITRE CORPORATION  
Bedford, Massachusetts  
Contract AF19(628)-2390 Project 750**

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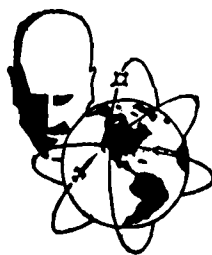
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#### FOREWORD

Except for minor editorial revisions, this report reproduces part of a paper which was presented to the Pulse Compression Symposium at RADC in October, 1962.

#### ACKNOWLEDGEMENT

I would like to credit Dr. R. Manasse and Messrs. E.L. Key and R.W. Jacobus for their previous work in tapped delay line techniques which led to this investigation. I would also like to thank Messrs. H.E.T. Connell and A.T. Kramer for their assistance with the computer programming which was an essential part of the study.

## ABSTRACT

This paper presents a method of linearly generating large time-bandwidth signals by tapped delay line filter synthesis techniques. The method is developed by an example wherein a filter with a constant amplitude and linear time delay transfer function of arbitrary time-bandwidth product is synthesized. Special emphasis is placed on equipment simplicity and ease of implementation. The essential elements employed by the technique are quartz delay lines, bandpass filters, bandpass phase shifters and resistive weighting networks. The technique does not make use of many elements which are found in most conventional pulse compression systems. In particular, dispersive networks, mixers, frequency synthesizers, and precision bandpass filters are not required. Consequently, the total number of components is small, even for time-bandwidth products of a few thousand, thereby making it feasible to obtain good peak signal to "hash" sidelobe levels at the matched filter output.

## PART II: TAPPED DELAY LINE SYNTHESIS OF LARGE TIME-BANDWIDTH SIGNALS

## INTRODUCTION

Nearly every aspect of pulse-compression system performance improves as the time-bandwidth-product increases. Simultaneously the problems of system complexity and component tolerance rapidly become more difficult. Using conventional lumped-constant filters, exceptional effort is required to obtain a TW-product of several hundred. The purpose of this paper is to present a method of linearly generating and receiving large TW-product signals by tapped delay linear filter synthesis techniques. Special emphasis is placed on equipment simplicity and ease of implementation. The essential elements employed by the technique are quartz delay lines, bandpass filters, bandpass phase shifters and resistive weighting networks. The technique does not make use of many elements which are found in most conventional pulse compression systems. In particular, dispersive networks, mixers, frequency synthesizers, and precision bandpass filters are not required. Consequently, the total number of components is small, even for TW-products of a few thousand, thereby making it feasible to obtain good peak-signal to "hash" sidelobe levels at the matched receiver output.

The paper consists of three sections: A discussion of a method of generating a large time-bandwidth signal with a tapped delay line, the results of a computer study concerned with minimizing the number of taps necessary to produce a given time-bandwidth product, and example designs of signals with TW-products of  $10^3$  and  $10^4$ . The last section contains a direct comparison of the TW= $10^3$  system and the 1000:1 linear-FM pulse compression system discussed in another paper<sup>1</sup> offered to this symposium.

## METHOD OF SYNTHESIS\*

In this paper the problem of generating a large TW-product signal will be considered to be equivalent to the problem of synthesizing a filter with a specified transfer function. A method of synthesis\* will be discussed by an example wherein a filter with a constant amplitude and linear time delay transfer function of arbitrary TW-product is synthesized. The specialization to this particular class of transfer function does not result in a loss of generality. The method may be applied to the problem of synthesizing an arbitrary transfer function.

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\*The synthesis method employed in this paper is based on and is an extension of tapped delay line techniques developed by Dr. R. Manasse<sup>2</sup>.



However, for an arbitrary function the resulting system will be considerably more complicated and the method loses some of its advantages over conventional lumped element techniques.

The problem to be considered is the synthesis of a linear filter which has the transfer function shown in Figure I.

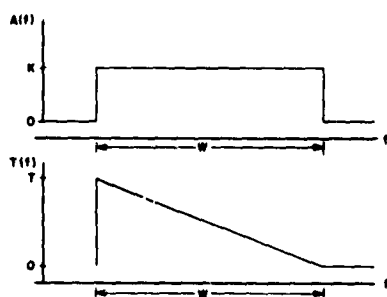


FIGURE I. DESIRED TRANSFER FUNCTION

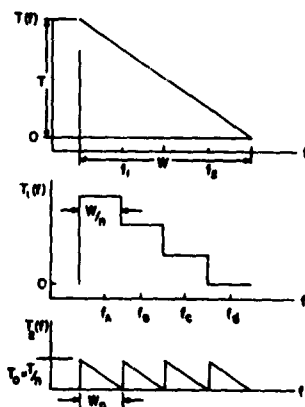


FIGURE II. DECOMPOSITION OF  $T(f)$

The amplitude characteristic is constant over the band of frequencies  $W$  cps. The group time delay characteristic is linear over the band  $W$  and dispersive by an amount  $T$  seconds. The time bandwidth product of this function is defined as  $TW$ .

The first step in the procedure is to resolve the desired function  $T(f)$  into the two functions  $T_1(f)$  and  $T_2(f)$  as shown in Figure II. The band  $W$  is divided into  $n$  equal intervals

$$W_0 = \frac{W}{n} \text{ cps.} \quad (1)$$

Each interval has

$$T_0 = \frac{T}{n} \quad (2)$$

seconds of dispersion. Then the total  $TW$ -product is

$$TW = n^2 T_0 W_0. \quad (3)$$

The function  $T_1(f)$  can, in principle, be obtained from a tapped delay line which has an appropriate ideal bandpass filter on each tap. The most efficient manner of obtaining the function  $T_1(f)$ , in terms of the number of taps for a given number of delay lines is illustrated in Figure III. This arrangement of filters and delay lines produces  $2^n$  taps from  $n$  lines.

The problem which remains to be solved in synthesizing the desired function  $T(f)$ , is the generation of the sawtooth type of time delay characteristic  $T_2(f)$ . This function is periodic in the band  $W$  with a period of  $W_0$  and it has  $T_0$  seconds of dispersion in each period.

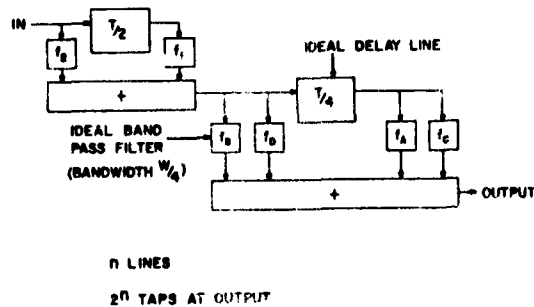


FIGURE III. TAPPED DELAY LINE REALIZATION OF  $T_1(f)$

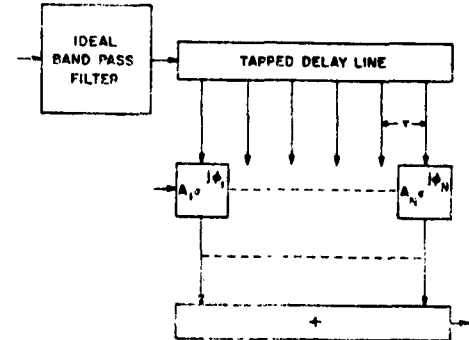


FIGURE IV.

Engelman<sup>3</sup> demonstrated in Part I of this paper that a band limited signal can be obtained from the cascade of an ideal bandpass filter and a tapped delay line as illustrated in Figure IV. The delay line shown has equally spaced taps, and amplitude and phase weights on each tap. The complex\* impulse response of this system is  $s_c(t)$ .

$$s_c(t) = B \sum_{k=1}^N A_k \frac{\sin \pi B(t-k\tau)}{\pi B(t-k\tau)} e^{j[\omega_0(t-k\tau) + \phi_k]} \quad (4)$$

Where:  $B$  is the bandwidth of the ideal filter in cps,  $\omega_0$  is the filter radian center frequency and  $\tau$  is the delay line tap spacing. The positive frequency portion of the transfer function of this system is  $S_c(\omega)$ , the Fourier transform of  $s_c(t)$ .

$$S_c(\omega) = \int_{-\infty}^{\infty} s_c(t) e^{-j\omega t} dt,$$

$$S_c(\omega) = \sum_{k=1}^N a_k e^{j\omega_k} e^{-j\omega k\tau}, \text{ for } (\omega_0 - \pi B) \leq \omega \leq (\omega_0 + \pi B) \quad (5)$$

$$S_c(\omega) = 0 \text{ elsewhere.}$$

\*The real time function is  $s(t) = 2 \operatorname{Re} [S_c(t)]$ .

$S_c(\omega)$  is a bandpass function with its band determined by the ideal filter.  $S_c(\omega)$  is a periodic function of frequency in the band of frequencies B, with a period given by the reciprocal of the tap spacing,

$$\text{period} = \frac{1}{T}.$$

However, the question remains, How many taps are necessary to produce  $T_2(f)$ , the function of interest? The right hand side of equation 5 is the complex Fourier series expansion of  $S_c(\omega)$  in the band of frequencies B. Then for the case

$$S_c(\omega) = T_2(f)$$

the necessary number of taps is identical to the number of Fourier coefficients in the expansion of  $T_2(f)$ . The problem is now reduced to the following question. Can  $T_2(f)$  be well represented by a small number of terms of its Fourier series expansion? If it can, then it can be synthesized by a small number of delay lines. Before continuing with the discussion it seems expedient to define some notations and summarize the development up to this point.

Since both of the time delay functions  $T_1(f)$  and  $T_2(f)$  can be obtained from tapped delay lines, it follows that the desired function  $T(f)$  can be obtained from the cascade of these two delay lines. In this paper, the delay line which generates the function  $T_2(f)$  will be called the synthesizing delay line and the line which generates  $T_1(f)$  will be called the recombination delay line.

The manner in which the recombination delay line converts the function  $T_2(f)$  into the total desired function  $T(f)$  is illustrated in Figure V. The function  $T_2(f)$  is the input to a filter bank which

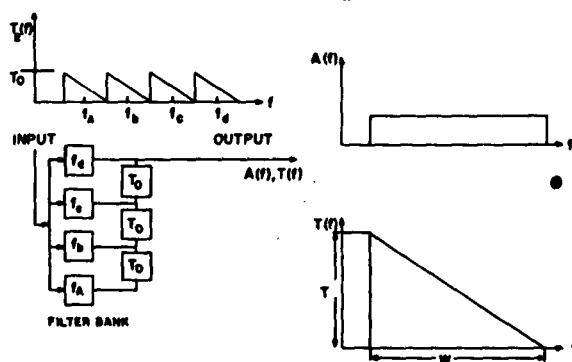


FIGURE V. RECOMBINATION PROCEDURE

contains as many filters as there are periods of  $T_2(f)$  in the band W. In the illustration of Figure V there are four. All are ideal bandpass filters centered at the frequencies  $f_a$ ,  $f_b$ ,  $f_c$ , and  $f_d$  as shown, with bandwidths  $W_0$ . The output of each filter versus frequency is a function with a rectangular amplitude, and a linear time delay. Henceforth in this paper, the time function which

appears at the output of each of these filters will be referred to as a subpulse and the filters will be called channel resolution filters.

Each subpulse has a time bandwidth product  $T_0 W_0$ . The subpulse with the highest center frequency  $f_d$  is not delayed, but goes directly to the output of the system. The pulse centered at  $f_c$  is delayed by an amount  $T_0$  and added to the first output pulse. The remaining pulses are differentially delayed by  $T_0$  and added in turn. The sum of these subpulses is an overall pulse whose spectrum amplitude versus frequency  $A(f)$  and time delay versus frequency  $T(f)$  are shown on the right hand side of Figure V.

In addition to the tapped recombination delay line a bank of ideal channel resolution filters is required. However, it is possible to replace this bank of ideal filters with two (or more) banks of non-ideal bandpass filters by requiring not one input periodic function but two (or more) periodic functions as illustrated in Figure VI. These

functions are periodic in both amplitude,  $A(f)$ , and time delay,  $T(f)$ , in the band  $W$ . As a consequence, the channel resolution filters need only be flat in a band  $W_0$  centered at a frequency, say  $f_a$ , and have the desired amount of rejection outside of a band  $3W_0$ . The combined result still produces the desired subpulses at the output of the non-ideal channel resolution filters and these subpulses can be recombined in a manner identical to the one demonstrated in Figure V.

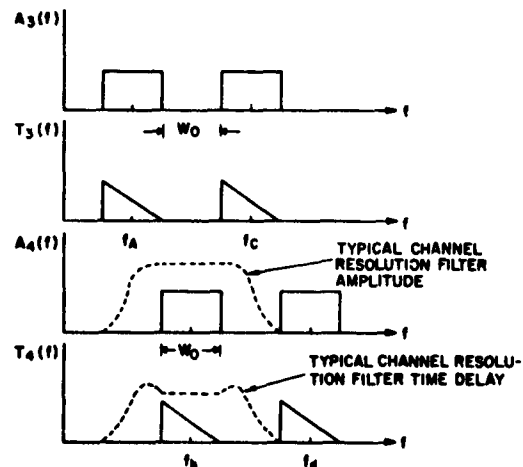


FIGURE VI.

It is now necessary to generate two periodic transfer functions.

Furthermore, each synthesizing delay line must accommodate enough Fourier series coefficients to well represent a periodic function of the type shown in Figure VI. The amplitude is unity for half of each period and is zero for the remaining half period. The time delay is linear for that part of each period where the amplitude is non-zero. Clearly, such a function will not be well represented by a small number of terms. It would seem that an amplitude which goes to zero in a more gradual manner would offer more promise. Such a compromise can easily be accommodated in the recombination process, for it merely means that the subpulses will overlap in frequency. Thus, the problem is finally reduced to the finding of a periodic function whose amplitude goes slowly to zero in such a manner that two adjacent subpulses add to unity in their overlap band, and whose time delay is linear when the amplitude is non-zero. A function which meets all of these requirements is shown in Figure VII.

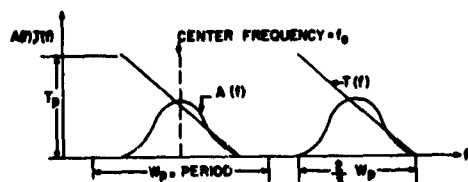


FIGURE VII. PERIODIC TRANSFER FUNCTION

Figure VII is a sketch of the periodic transfer function which will be discussed for the remainder of this paper. This function is the building block upon which the success of this particular synthesis method depends. The amplitude characteristic  $A(f)$  is defined as

$$A(f) = \begin{cases} 0 & , \text{ for } -\frac{1}{2} \leq \frac{f-f_0}{W_p} < -\frac{1}{3} \\ 1 + \cos 3\pi \frac{(f-f_0)}{W_p} & , \text{ for } -\frac{1}{3} \leq \frac{f-f_0}{W_p} \leq \frac{1}{3} \\ 0 & , \text{ for } \frac{1}{3} < \frac{f-f_0}{W_p} \leq \frac{1}{2} \end{cases} \quad (6)$$

The time delay characteristic is

$$T(f) = \begin{cases} \text{Undefined} & \\ \frac{T_p}{2} \left[ 1 - 3 \frac{(f-f_0)}{W_p} \right] & , \text{ same as above.} \\ \text{Undefined} & \end{cases}$$

Note that the time delay need only be controlled over that portion of the period where the amplitude is non-zero. Therefore,  $T_p$  need only be

$$T_p = 2T_0 \quad (7)$$

as shown in Figure VIII. However, due to the zero amplitude region in each period, the period,  $W_p$ , must be

$$W_p = 3W_0 \quad (8)$$

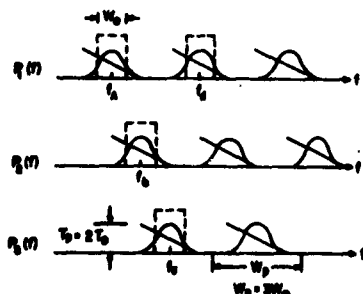


FIGURE VIII. THREE SHIFTED PERIODIC FUNCTIONS

The use of this particular function makes it necessary to generate three periodic functions and to use three separate banks of channel resolution filters. The three periodic functions are shifted in frequency with respect to each other by  $W_0$  cps as illustrated in Figure VIII. That is, the center frequencies of two adjacent subpulses differ by

$$W_0 = \frac{W}{3} \text{ cps.}$$

This shift assures that only two subpulses overlap in any given region. The shape of the subpulses is such that they add to a constant in the overlap region. This process of addition can be seen from Figure VIII and may be demonstrated in the following manner. Neglect phase terms, that is, assume the subpulses have been properly delayed so that they have the same time delay curves in the overlap frequency region. Then let

$$|P_1(f)| = 1 + \cos 3\pi \frac{(f-f_A)}{W_p}, \text{ for } f_A \leq f \leq (f_A + \frac{W}{3})$$

and

$$|P_2(f)| = 1 + \cos 3\pi \frac{(f-f_B)}{W_p}, \text{ for } (f_B - \frac{W}{3}) \leq f \leq f_B,$$

where:

$$f_B = f_A + \frac{W}{3},$$

so that

$$|P_2(f)| = 1 - \cos 3\pi \frac{(f-f_A)}{W_p}, \text{ for } f_A \leq f \leq (f_A + \frac{W}{3}).$$

In like manner

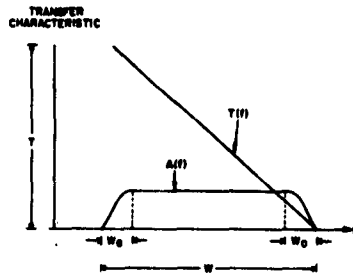
$$|P_3(f)| = 0 \quad \text{for } f_A \leq f \leq (f_A + \frac{W}{3}).$$

Similar conditions apply in all other regions of the frequency which are in the band of interest. Then the sum of these adjacent channels gives,

$$|P_1(f) + P_2(f) + P_3(f)| = 2, \text{ for } f_A \leq f \leq f_A + \frac{W}{3}.$$

If K periods of each of the three periodic functions are used, then 3K subpulses are obtained. The over-all pulse bandwidth is defined as that frequency band over which the total time delay is linear, as shown in Figures I and IX.

Consequently, the band  $W$  is given by



$$\begin{aligned} W &= \left(K + \frac{1}{3}\right) W_p \\ &= (3K + 1) \frac{W_p}{3} \\ &= (3K + 1) W_0 \end{aligned}$$

FIGURE IX. SYSTEM TRANSFER FUNCTION

Combining this result with equation (1) shows that  $n$  and  $K$  are related by

$$n = 3K + 1$$

Substitution of equations (7) and (8) into equation (1) yields,

$$TW = n^2 \frac{T W_p}{6}$$

Combining this result with equation (5) gives,

$$TW = (3K + 1)^2 \frac{T W_p}{6} \quad (10)$$

Equation (10) relates the overall  $TW$ -product and the  $TW$ -product of one period of the periodic transfer function defined in equation (6) when  $K$  periods of the function are employed.

The manner in which the  $3K$  subpulses are generated and recombined is illustrated in Figure X. Three tapped delay lines are used to obtain the three periodic transfer functions  $P_1(f)$ ,  $P_2(f)$  and  $P_3(f)$ . Each filter bank contains  $K$  channel resolution filters. The subpulses at the output of the channel resolution filters are recombined in a manner identical to that discussed previously. The over-all transfer function is shown in Figure IX. The group delay characteristic varies linearly by  $T$  seconds over the band  $W$ . The amplitude is flat over the band except for a region of  $W_0$  cps at each end, where it falls off as the  $[1 + \cosine]$  amplitude of a single channel.

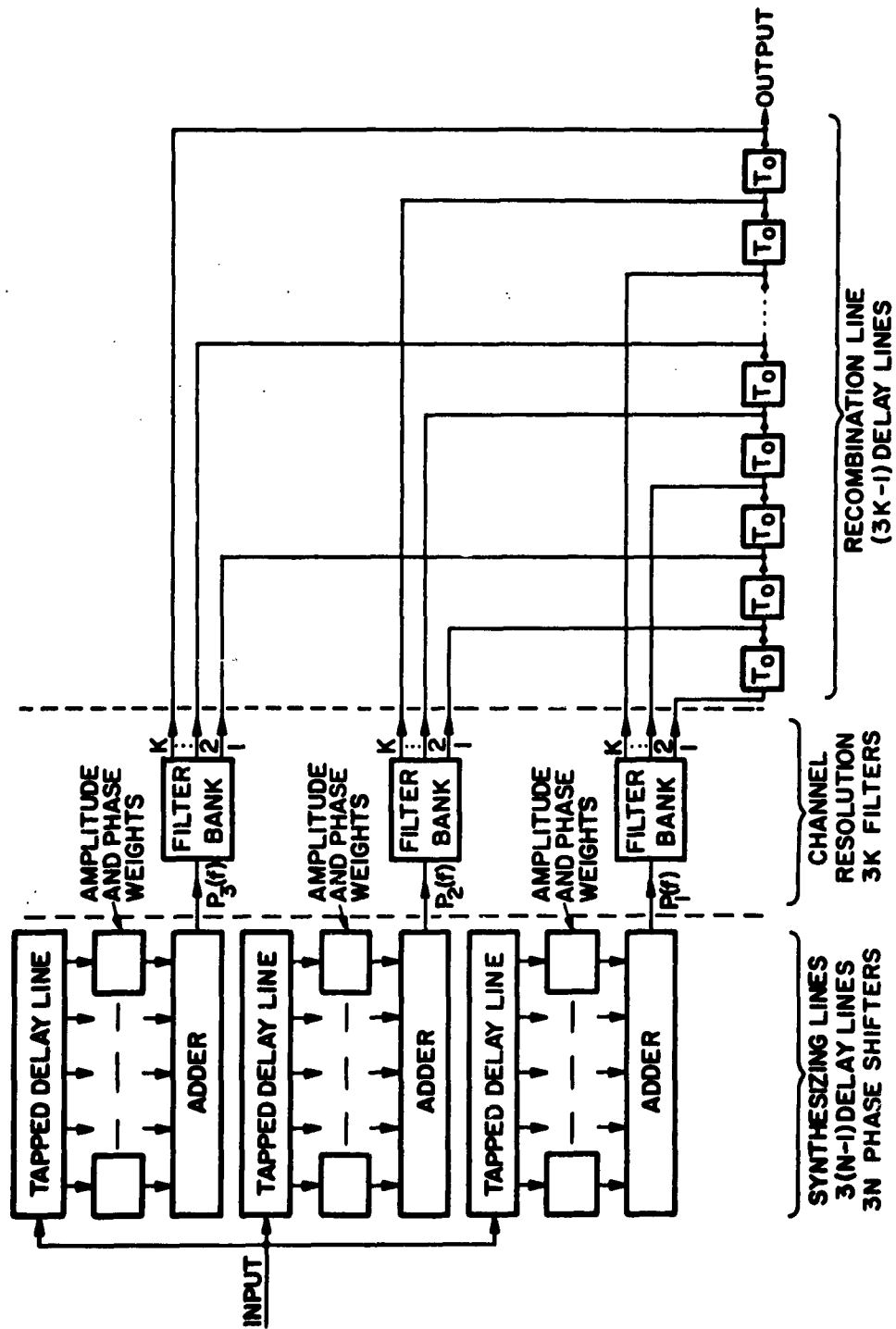


FIGURE X. ILLUSTRATIVE BLOCK DIAGRAM



Considerable reduction of the complexity of the system shown in Figure X is possible. The system shown contains three separate synthesizing tapped delay lines, each of which contain  $N$  taps, giving a total of  $3(N-1)$  delay lines. However, the three synthesizing lines have transfer functions which differ only by a frequency shift. Consequently, the three synthesizing lines are identical except for the phase weights on their taps (i.e., the tap spacing and the amplitude weights are the same). Thus, the three tapped delay lines may be replaced by one tapped delay line with the three buffer amplifiers on each tap, thereby saving  $2(N-1)$  delay lines, as illustrated in Figure XI.

Of course, it is still necessary to construct three separate sets of weighting networks. But, it is not necessary to construct a bandpass phase shifter for each tap weight. The complex weighting network can be replaced by two real quadrature weighting networks and a bandpass  $90^\circ$  phase shifter. This equivalence is illustrated in Figure XII.

Further, simplification of the system illustrated in Figure X is possible. The recombination tapped delay line is made up of  $(3K-1)$  delay lines. The arrangement shown in Figure XIII performs the recombination of the subpulses with only  $K+1$  delay lines, a saving of  $2(K-1)$  delay lines.

The reduced version of the total system is illustrated in Figure XIV. The outputs of the taps on the one synthesizing tapped delay line are buffered so that three different periodic functions can be obtained from one tapped line and three weighting networks. Each weighting network consists of two sets of real weights and one  $90^\circ$  bandpass phase shifter. Two of the resulting periodic functions are delayed, one is not, all enter their separate banks of channel resolution filters. Because of the delay lines prior to the channel resolution filters the subpulses can be added in triplets. These summed triplets are then recombined in a manner identical to that discussed previously. The over-all transfer function of the entire system is shown in Figure IX. The group delay characteristic varies linearly by  $T$  seconds over the band  $W$ . The amplitude is flat except for a region of  $W_0$  cps at each end of the band, where it falls off as the  $[1 + \cos]$  amplitude of a single channel.

The synthesis procedure is complete. However, the question remains as to how many taps are necessary on the synthesizing delay line in order to produce the particular periodic function employed. Toward this end, a computer study was conducted and the results are presented in the next section.

#### COMPUTER STUDY

This section treats the following problem. For the periodic transfer function defined in Equation (6), how many taps must be used on the synthesizing delay line? Clearly the answer depends upon the allowable approximation error and the time-bandwidth product  $T_p W_p$  of

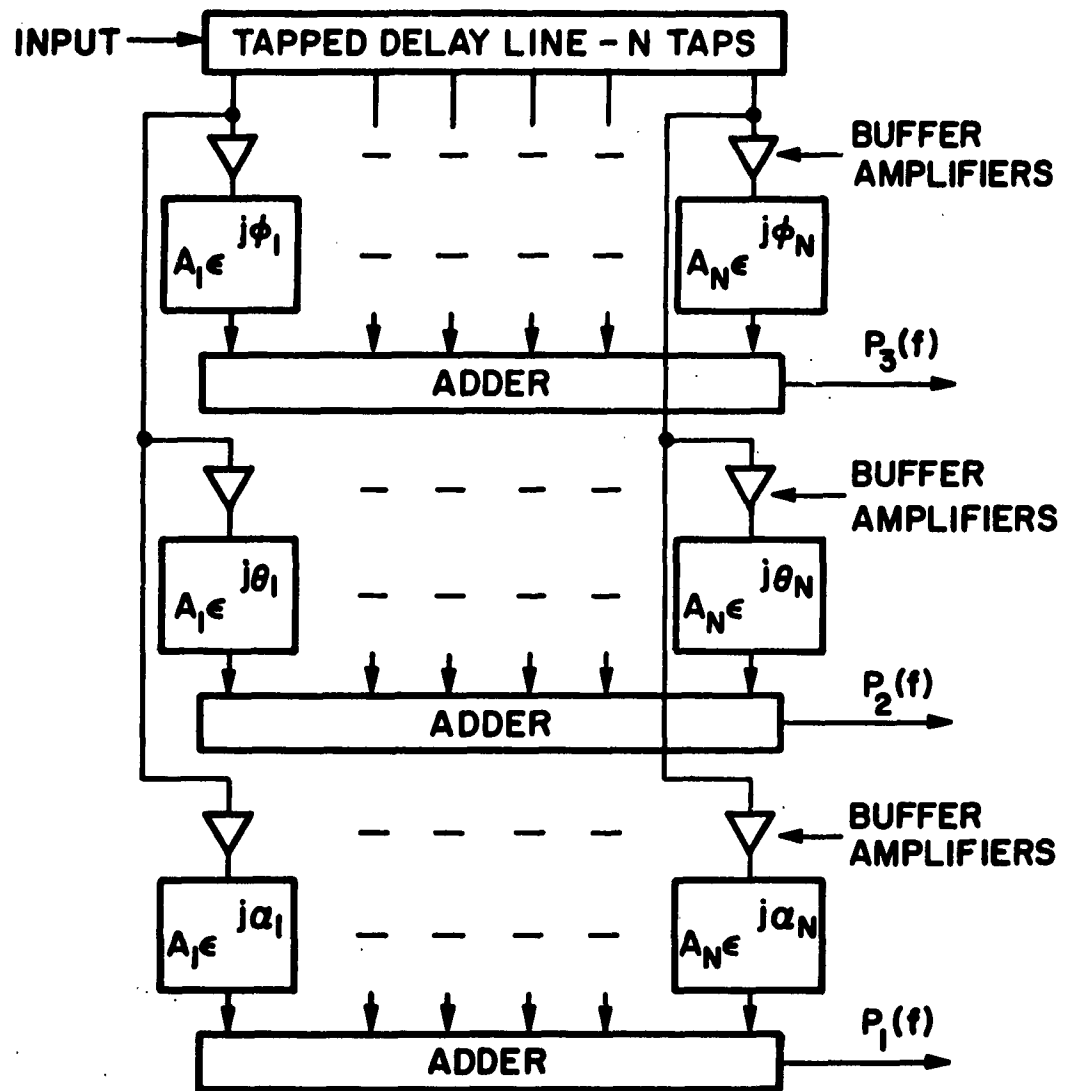


FIGURE XI

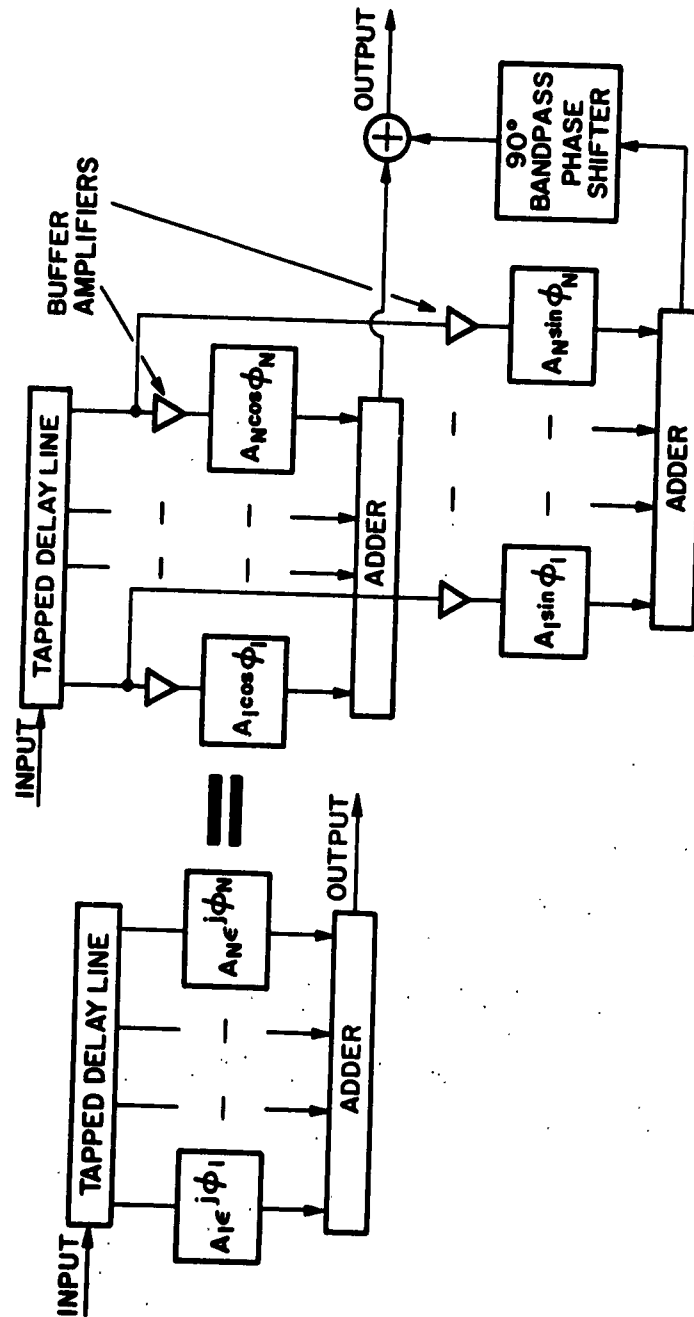


FIGURE XII

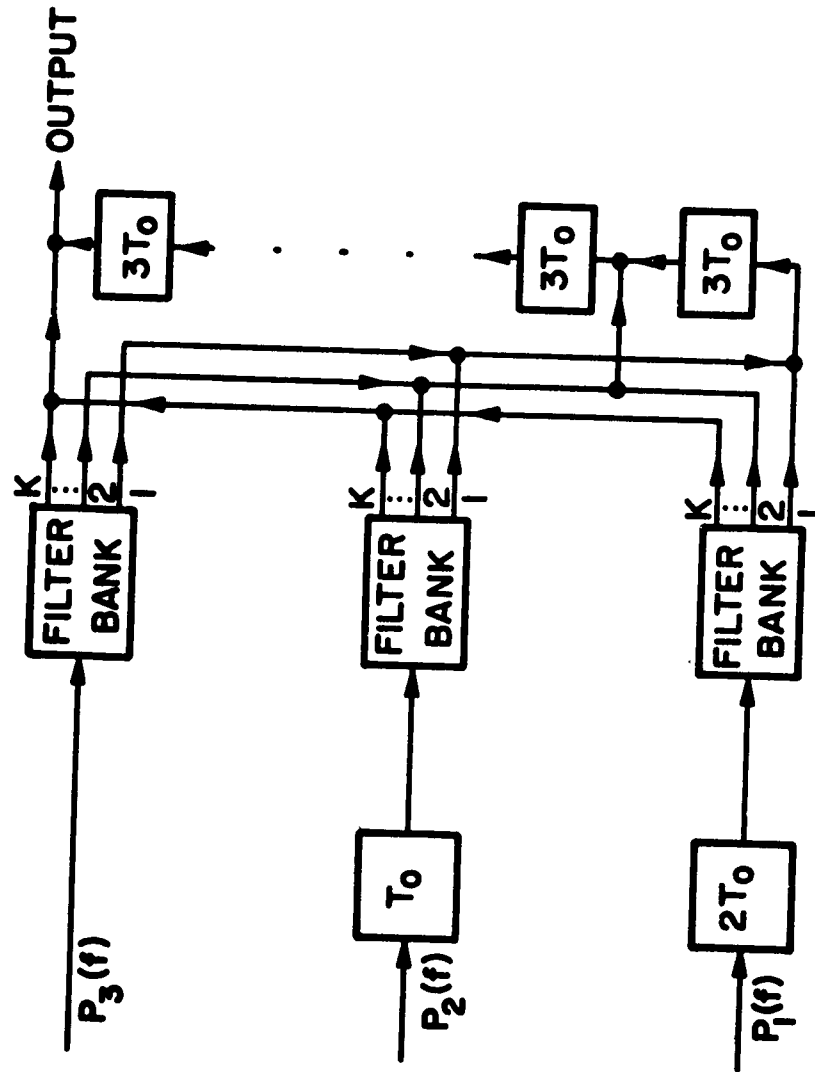


FIGURE XIII. SIMPLIFIED RECOMBINATION PROCEDURE

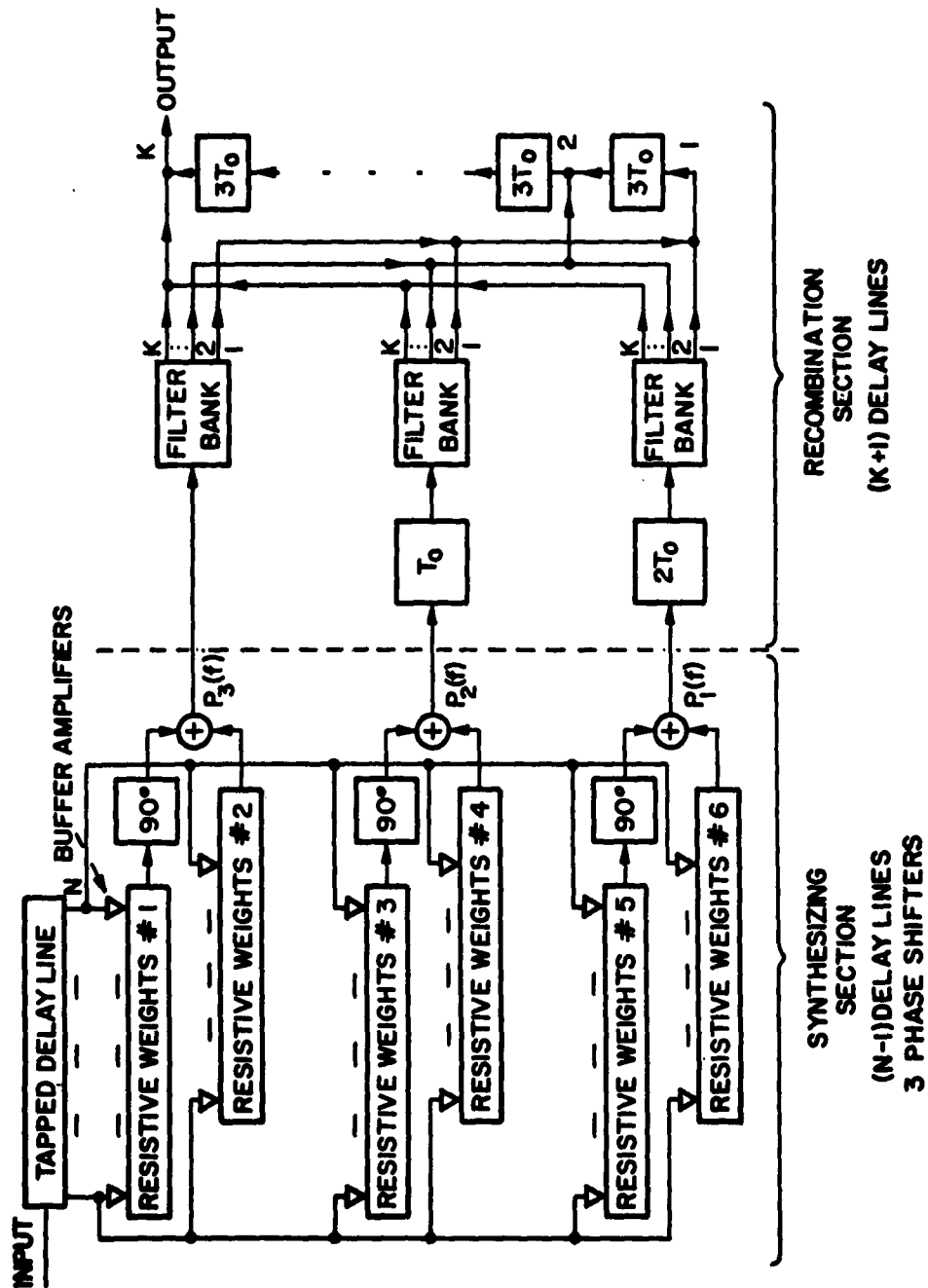


FIGURE XIV. SYSTEM BLOCK DIAGRAM

each period of the function. The errors which are of interest in the application at hand are as follows:

$\Delta A$  = the ripple in the overall recombined system transfer function amplitude

$\Delta \phi$  = ripple in the overall recombined system transfer phase characteristic

P = period of  $\Delta \phi$  and  $\Delta A$  in cps.

These errors are a result of taking only a finite number of taps on the synthesizing delay line. Such errors will produce both a ripple in the envelope of "transmitted" pulse of the system and "spurious sidelobes" in the matched receiver output. Some example results of the computer study are presented in TABLE I.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
TW	$T \frac{W}{P}$	N	K	K+1	N+K	$\frac{\Delta A}{\%}$	$\Delta \phi$ ± Degrees
1,000	9.6000	16	8	9	24	1.09	0.58
1,000	3.7500	11	13	14	24	0.61	0.80
10,000	19.8347	25	18	19	43	1.00	0.56
10,000	13.3660	20	22	23	42	0.92	0.44
10,000	9.6138	16	26	27	42	1.10	0.58
10,000	3.7200	11	42	43	53	0.20	0.80

$$P = W_p / 3 \text{ cps}$$

TOTAL NUMBER OF CHANNEL SELECTION FILTERS = 3K

NUMBER OF PERIODS USED = K

NUMBER OF TAPS ON SYNTHESIZING DELAY LINE = N

NUMBER OF RECOMBINATION LINES = K + 1

TOTAL NUMBER OF SYSTEM LINES = N + K

TABLE I - DESIGN RESULTS

The computer calculations assume ideal delay lines and tap weights and are based upon the allowable approximation errors shown in the Table. The calculations consisted of the following:

1. Make a Fourier series expansion of the periodic transfer function defined in equation (6) for various values of  $T W_p$ .
2. Truncate the expansion so that it contains only N terms.
3. Using the N Fourier coefficients as tap weights, resynthesize the three approximate periodic transfer functions.
4. Assume ideal channel resolution filters and recombine K periods of each function to obtain the overall transfer function.

It is not necessary to assume ideal channel resolution filters. Non-ideal filters can be amplitude and phase-corrected by simply pre-distorting the desired periodic function before making the Fourier series expansion. The pre-distortion will result in slightly different values for the tap weights on the synthesizing line. However, for simplicity this step was omitted in this initial theoretical study. It would, of course, be included in the design of a particular system.

The fourth column of Table I gives the number of channel resolution filters in each of the three filter banks. The sixth column gives the total number of delay lines in the system necessary to produce a function whose TW-product is given in the first column and whose amplitude and phase ripples are given in columns 7 and 8. The various examples are presented to illustrate the flexibility in apportioning the total number of delay lines in the system to the two separate tapped delay lines. The flexibility has practical design importance in that the various delay lines have different specifications. For example, the lines used in the synthesizing delay line have the total system bandwidth but in general have short delay. The opposite is true of some of the recombination lines. It should be noted, concerning design flexibility, that many other arrangements of the various delay lines<sup>1,2</sup> can be employed. For example, the "recombination" tapped delay line can precede the synthesizing lines. Regardless of the order in which the various units are arranged, the periodic nature of a tapped delay line remains as the dominant factor in producing large TW-product signals with a relatively small number of components. The particulars of a typical calculation will now be considered in an example design.

## EXAMPLE DESIGN

The purpose of this design example is two-fold: first to clarify the method of synthesis which has been developed and second to demonstrate the simplicity of the resulting equipment. Consequently, the system numbers have been chosen to permit a direct comparison of this system and the system which is discussed by R. W. Jacobus in another paper<sup>1</sup> presented to this symposium. For clarity in this discussion, Jacobus' system will be called the "ten-channel" system.

The desired TW-product is  $10^3$  and the recombination delay line contains nine (9) lines. In this respect the system is identical to the ten-channel (nine line) system discussed by Jacobus.

The time-bandwidth-product of one period of the periodic transfer function is computed from equation (10):

$$T_p W_p = \frac{6 TW}{(3K+1)^2}$$

K = one less than number of recombination delay lines

$$K = 9 - 1 = 8$$

$$T_p W_p = \frac{6 \times 10^3}{(25)^2} = 9.6$$

Let,  $T = 1000 \mu\text{seconds}$  and  $W = 1 \text{ mcps}$ .

$$\text{Then, } T_p = \frac{2T}{(3K+1)} = \frac{2 \times 10^3}{25}$$

$$= 80 \mu\text{sec}$$

$$W_p = \frac{3W}{3k+1} = 120 \text{ Kcps}$$

Table II gives a list of the resulting system design parameters.



$$T_{pp} W_p = 9.6$$

$N = 16$  = number of taps on the synthesizing delay line

$MK = 24$  total number of channel resolution filters

$M = 9$  = number of recombination delay lines

$MK = 24$  = total number of system delay lines

$$\Delta A = \pm 1.09\%$$

$$\Delta \phi = \pm 0.579 \text{ Degrees}$$

TABLE II - SYSTEM PARAMETERS

Although there are 16 taps on the synthesizing delay line there must be three separate sets of weights, or a total of 48 complex weights. This does not imply, however, that 48 bandpass phase shifters must be used! Instead each of the three periodic transfer functions is obtained from 32 real weights and one  $90^\circ$  bandpass phase shifter. Thus, the total system has 96 resistive (real) weights and three bandpass  $90^\circ$  phase shifters.

Figure XV shows one period of the error in the system transfer amplitude characteristic. Figure XVI shows a similar curve of phase error. These theoretical errors are better than the corresponding theoretical errors in "ten-channel" system discussed by Jacobus. In addition, the system developed here should produce better practical fabrication errors because

of the tremendous decrease in equipment complexity compared to the "ten-channel" system. For example, the additional 15 delay lines, the channel resolution filters and the tap weights replace the precise channel amplitude shaping i-f strips, the allpass networks, the local oscillators, mixers

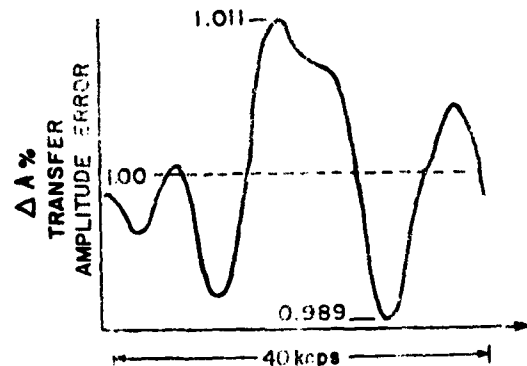


FIGURE XV. TRANSFER AMPLITUDE ERROR

and sideband selection filters of the "ten-channel" system. Of course, this reduction of the number of components is made possible because the quartz delay lines are inherently large TW-product devices and are capable of replacing many small TW-product components. The block diagram of the example system will be totally similar to the one shown in Figure XIV.

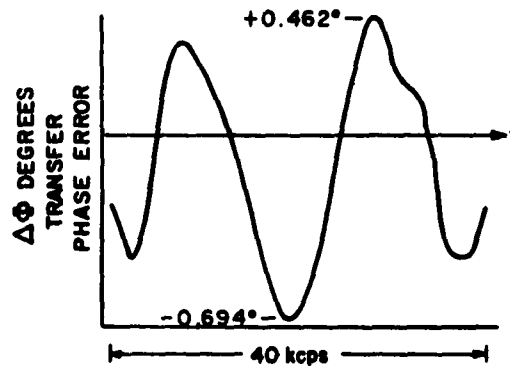


FIGURE XVI.  $\Delta\phi$  DEGREES PHASE ERROR

#### CONCLUSION

A theoretical method of tapped delay line filter synthesis has been presented. It has been shown that a linear filter with a large TW-product impulse response can be synthesized with a small number of components. The reduction in system complexity, compared to conventional techniques, is made possible by the use of inherently large TW-product devices, viz. quartz delay lines. Since the method depends primarily upon a small number of stable, accurate quartz delay lines the attainment of extremely low sidelobe levels at the output of the matched receiver is feasible. Emphasis has been placed upon equipment simplicity and ease of implementation. Of additional interest is the growth of system complexity with TW-product. Although 24 delay lines are necessary to produce a TW-product of  $10^3$  only 42 lines are needed to generate  $10^4$  - an increase of less than two in complexity. It is believed that this method of signal synthesis will prove to be highly practicable, especially in view of the promising results attained in the tapped delay line system discussed by Jacobus.

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<p>Hq. ESD, L.G. Hanscom Field, Bedford, Mass.</p> <p>Rpt. No. ESD-TDR-63-180. TAPPED DELAY LINE SYNTHESIS OF LARGE TIME-BANDWIDTH SIGNALS. Final report, June 1963, 23p. incl. illus., 3 refs.</p> <p>Unclassified Report</p> <p>This paper presents a method of line-early generating large time-bandwidth signals by tapped delay line filter synthesis techniques. The method is developed by an example wherein a filter with a constant amplitude and linear time delay</p>	<p>1. Pulse compression</p> <p>2. Delay lines</p> <p>3. Bandpass filters</p> <p>4. Phase shifters</p> <p>I. Project No. 750</p> <p>II. AFI9(628)-2390</p> <p>III. The MITRE Corporation, Bedford, Mass.</p> <p>IV. Haggarty, R.D.</p> <p>V. TM-3535</p>	<p>Hq. ESD, L.G. Hanscom Field, Bedford, Mass.</p> <p>Rpt. No. ESD-TDR-63-180. TAPPED DELAY LINE SYNTHESIS OF LARGE TIME-BANDWIDTH SIGNALS. Final report, June 1963, 23p. incl. illus., 3 refs.</p> <p>Unclassified Report</p> <p>This paper presents a method of line-early generating large time-bandwidth signals by tapped delay line filter synthesis techniques. The method is developed by an example wherein a filter with a constant amplitude and linear time delay</p>	<p>1. Pulse compression</p> <p>2. Delay lines</p> <p>3. Bandpass filters</p> <p>4. Phase shifters</p> <p>I. Project No. 750</p> <p>II. AFI9(628)-2390</p> <p>III. The MITRE Corporation, Bedford, Mass.</p> <p>IV. Haggarty, R.D.</p> <p>V. TM-3535</p>
<p>Hq. ESD, L.G. Hanscom Field, Bedford, Mass.</p> <p>Rpt. No. ESD-TDR-63-180. TAPPED DELAY LINE SYNTHESIS OF LARGE TIME-BANDWIDTH SIGNALS. Final report, June 1963, 23p. incl. illus., 3 refs.</p> <p>Unclassified Report</p> <p>This paper presents a method of line-early generating large time-bandwidth signals by tapped delay line filter synthesis techniques. The method is developed by an example wherein a filter with a constant amplitude and linear time delay</p>	<p>1. Pulse compression</p> <p>2. Delay lines</p> <p>3. Bandpass filters</p> <p>4. Phase shifters</p> <p>I. Project No. 750</p> <p>II. AFI9(628)-2390</p> <p>III. The MITRE Corporation, Bedford, Mass.</p> <p>IV. Haggarty, R.D.</p> <p>V. TM-3535</p>	<p>Hq. ESD, L.G. Hanscom Field, Bedford, Mass.</p> <p>Rpt. No. ESD-TDR-63-180. TAPPED DELAY LINE SYNTHESIS OF LARGE TIME-BANDWIDTH SIGNALS. Final report, June 1963, 23p. incl. illus., 3 refs.</p> <p>Unclassified Report</p> <p>This paper presents a method of line-early generating large time-bandwidth signals by tapped delay line filter synthesis techniques. The method is developed by an example wherein a filter with a constant amplitude and linear time delay</p>	<p>1. Pulse compression</p> <p>2. Delay lines</p> <p>3. Bandpass filters</p> <p>4. Phase shifters</p> <p>I. Project No. 750</p> <p>II. AFI9(628)-2390</p> <p>III. The MITRE Corporation, Bedford, Mass.</p> <p>IV. Haggarty, R.D.</p> <p>V. TM-3535</p>

<p>transfer function of arbitrary time-bandwidth product is synthesized. Special emphasis is placed on equipment simplicity and ease of implementation. Essential elements employed by the technique are quartz delay lines, bandpass filters, bandpass phase shifters and resistive weighting networks. The technique does not make use of many elements found in most conventional pulse compression systems. Dispersive networks, mixers, frequency synthesizers, and precision bandpass filters <input type="radio"/> are not required.</p>	<p>VI. In DDC collection</p>	<p>transfer function of arbitrary time-bandwidth product is synthesized. Special emphasis is placed on equipment simplicity and ease of implementation. Essential elements employed by the technique are quartz delay lines, bandpass filters, bandpass phase shifters and resistive weighting networks. The technique does not make use of many elements found in most conventional pulse compression systems. Dispersive networks, mixers, frequency synthesizers, and precision bandpass filters <input type="radio"/> are not required.</p>	<p>VI. In DDC collection</p>
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